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EXAMINATION OF A SOLID STAMP ON AN ELASTIC FOUNDATION

СЕРПІМДІ НЕГІЗДЕГІ ҚАТТЫ ШТАМПТЫ ЗЕРТТЕУ

ИССЛЕДОВАНИЕ ЖЕСТКОГО ШТАМПА НА УПРУГОМ ОСНОВАНИИ

Abstract. The paper considers obtaining results of calculating the normal stress, vertical displacement, deflection and reactive pressure for a solid stamp on an elastic foundation. The values of these calculations are used in solving problems of elasticity theory, but in practice it is difficult to obtain the results of these problems. According to the Sadovsky method, the value of the reactive pressure on an elastic half-plane foundation under a solid stamp tends to infinity. Calculations performed for a solid stamp on an elastic base are poorly studied, but are widely used in modern construction, and improvement of its calculation methods is mandatory. Therefore, the development of new, simple analytical and numerical improved methods for solving these problems is relevant. The paper defines the values of the deflection function for soil foundation with different modulus of elasticity. As a result, the bearing property of the foundation decreases, the elastic soil foundation tends to perceive heavy loads, affects its shrinkage and displacement. In addition, a change in the values of normal stress and vertical displacement was detected due to the transformation of the l/h ratio lying outside the stamp. The maximum values of normal stress and vertical displacement in the specified intervals and the reactive pressure of the stamp under the action of a uniformly distributed load are calculated. The results of these calculations play an important role in the field of construction mechanics, that is, simplify the process of analysis and design of structures. The development of this improved equation opens up new opportunities for further research and implementation into engineering practice, and is also used to calculate a beam on an elastic foundation.

Keywords: elastic foundation, stamp, normal stress, vertical displacement, deflection, reactive pressure.

Аңдатпа. Мақалада серпімді негіздегі қатты штамп үшін нормалдық кернеу, тік жылжу, майысу және реактивтік қысым нәтижелерін алу қарастырылған. Бұл есептеулердің мәндері серпімділік теориясының есептерін шешуде қолданылады, бірақ іс жүзінде алынған нәтижелер өте күрделі болып келеді. Садовский әдісі бойынша қатты штамп астындағы серпімді жартылай негізде пайда болатын реактивтік қысым штамптың периметрі бойынша шексіздікке ұмтылады. Серпімді негіздегі қатты штамп үшін орындалатын байланыс есептеулері аз зерттелген, бірақ қазіргі заманғы құрылыста кеңінен қолданылады және оны есептеу әдістерін жетілдіру міндетті. Сондықтан осы мәселелерді шешудің жаңа, қарапайым аналитикалық және сандық жетілдірілген әдістерін әзірлеу өзекті мәселе болып табылады. Мақалада әртүрлі серпімділік модулі бар топырақты негіздер үшін, майысу функциясының мәндері анықталды. Нәтижесінде негіздің

көтерілістік қасиеті төмендегенін көрсетеді, серпімді топырақ негізі үлкен жүктемелерді қабылдау қасиеті, оның шөгуді мен жылжуына әсерін тигізеді. Сонымен қатар, штамптан тыс жатқан l/h қатынасын өзгерту арқылы нормальді кернеу мен тік жылжу мәндерінің өзгерісі анықталды. Нормальді кернеу мен тік жылжудың берілген аралықтардағы ең үлкен мәндері және жинақталған күш әсер еткенде штамптың реактивтік қысымы есептелінді. Мақалада алынған нәтижелер құрылыс механика саласында маңызды болып табылады, яғни құрылымдарды талдау және жобалау процесін жеңілдетеді. Бұл жетілдірілген тәсілді әзірлеу кейінгі зерттеулер мен инженерлік тәжірибеге енгізудің жаңа мүмкіндіктерін ашады және оны серпімді негіздегі арқалықты есептеу саласында қолданылады.

Түйін сөздер: серпімді негіз, штамп, нормальді кернеу, тік жылжу, майысу, реактивтік қысым.

Аннотация. В статье рассматривается получение результатов расчета нормального напряжения, вертикального перемещения, изгиба и реактивного давления для жесткого штампа на упругом основании. Значения этих расчетов используются при решении задач теории упругости, однако на практике получения результатов этих задач является сложным. По методу Садовского значение реактивного давления на упругом полуплоском основании под жестким штампом стремится к бесконечности. Расчеты, выполняемые для жесткого штампа на упругом основании малоизучены, но широко используются в современном строительстве, и усовершенствование методов его расчета является обязательным. Поэтому разработка новых, простых аналитических и численных усовершенствованных методов решения этих задач является актуальным. В статье определены значения функции изгиба для грунтовых оснований с различным модулем упругости. В результате несущее свойство основания снижается, упругое грунтовое основание имеет свойство воспринимать большие нагрузки, влияет на его усадку и перемещение. Кроме того, обнаружено изменение значений нормального напряжения и вертикального перемещения за счет преобразования отношения l/h , лежащего вне штампа. Рассчитаны максимальные значения нормального напряжения и вертикального перемещения в заданных интервалах и реактивное давление штампа при действии равномерно распределенной нагрузки. Результаты указанных расчетов играют большую роль в области строительной механики, то есть упрощают процесс анализа и проектирования конструкций. Разработка данного усовершенствованного уравнения открывает новые возможности для последующих исследований и внедрения в инженерную практику, также используется для расчета балки на упругом основании.

Ключевые слова: упругое основание, штамп, нормальное напряжение, вертикальное перемещение, изгиб, реактивное давление.

Introduction. The theory of calculating beams on an elastic foundation is important in construction practice. The problems associated with the study of structures lying on an elastic foundation represent one of the most urgent, complex interesting tasks of construction mechanics. Recently, attention to these tasks has been increasing. On the one hand, this is due to the urgent needs of engineering practice, and on the other hand, to the development and improvement of calculation methods. When calculating a beam on an elastic foundation, first we establish a calculation model of the beam and consider the conditions of its contact with the foundation, and then we calculate and compare the calculation options (Khryanina, 2016).

Two solutions of the theory of elasticity have become important for the calculation of beams on an elastic foundation. Boussinesq's solution for a solid elliptical stamp on an elastic half-space in its special case – for a solid centrally loaded round stamp - has found wide application for the calculation of round solid plates. The second solution – for a solid strip in the conditions of a flat problem – belongs to Sadovsky. The obtained formulas and diagrams of reactive pressures are almost identical (Tsvei, 2014). L.G. Petrosyan's paper (Yaretskaya, 2018) provides a numerical solution to the contact problem for a solid stamp with a flat sole pressed without friction into an elastic foundation described by a generalized model. The contact problem is formulated in the form of paired integral equations, and the proposed model allows us to obtain pressure plots for an elastic half-space.

Spatial contact problems with an unknown contact area are studied for an elastic layer of finite

thickness, into one face of which two identical elliptical solid stamps are symmetrically pressed. The problems are reduced to an integral equation regarding the contact pressure. As a result, the contact pressure is decomposed in a series according to the degrees of a small parameter characterizing the relative thickness of the elastic layer (Balabušić, Folić & Ćorić, 2019).

When solving problems of elasticity theory for a half-space, integrations of cumulative differential equations are used, but in practice, this method turns out to be very difficult. Therefore, a semi-reversible method was proposed to obtain the final solution, that is the reaction of an elastic half-space under a centrally loaded rectangular stamp. The reaction of the elastic half-space and the reactive pressure along the perimeter of the stamp takes the value to infinity (Bosakov, 2019).

The effect of a uniformly distributed load when calculated according to the theory of elasticity, the reactive pressures are uniform only for an infinitely flexible band ($t \rightarrow \infty$). As the flexibility index decreases, the unevenness of the plot increases with increasing pressure towards the edges. At $t=0$, so for a solid strip, the reactive pressures are distributed according to Sadovsky's formula, and integrating this equation at the free edges of the strip, we obtained the equations for a solid stamp (Ledenov, Monastirev et al., 2016).

In the field of construction mechanics of contact interactions, non-stationary contact problems are the least studied. These tasks are extremely important for various sectors of the national economy, such as geophysics, seismology, acoustics, vibro-seismic exploration, foundation engineering, military industry, etc. (Li, Arutiunian, Kuznetsova & Fedotenkov, 2020). To date, there is only a limited range of works devoted to the study of the processes of non-stationary contact interaction for a solid stamp and an elastic half-space. In one of the papers, a formulation is given and a method is developed for solving new planar non-stationary contact problems for absolutely solid stamps and an elastic half-space containing a recessed cavity with a smooth boundary of arbitrary geometry. A resolving boundary integral equation is constructed using Green's functions for an elastic half-plane. In the work of the following authors (Arutyunyan, Kuznetsova & Fedotenkov, 2020), three solutions to the contact problem are considered: free sliding, solid coupling and related contact. The method of solving the problem is constructed using complex boundary integral equations using the dynamic mutual work theorem. The method and solution algorithm developed by the authors allow it possible to study the processes of non-stationary contact interaction of solids with an elastic half-space.

The calculation of the elastic precipitation of solid rectangular foundations is one of the most important tasks in classical mechanics for solving the precipitation of shallow foundations. To calculate them, an analysis of the method of calculating the elastic precipitation of a solid foundation was proposed. In the proposed pressure problems, when using a reference pressure, the calculated value turns out to be unfavorable, which explains the complexity of the analytical evaluation of the method. The methods of solving the problem proposed by the authors are based on the concept of an equivalent shape, which replaces any solid rectangular foundation with a foundation on the concept of an equivalent shape. The analysis showed that an equivalent ellipse gives satisfactory results only if the condition of an even area, that is, an even perimeter length, is fulfilled (Lysandros, Elias, 2020).

Contact problems for an elastic strip on a solid foundation have been studied a lot less, but are widely used in modern construction and improving its calculation methods is a necessary engineering task. To construct a matrix of vertical displacements of the points of the end of the half-strip from the concentrated force, variational-difference methods are used. Based on these movements, without taking into account friction in the contact zone, the tasks for a stamp of different sizes pressed against the end of the strip are solved. The results of the distribution of contact stresses and deflection moments, the vertical displacements almost coincide with similar calculations for an elastic half-plane (Bosakov, 2023).

The above works are distinguished by the complexity of mathematical calculations to find a solution. The solution of the contact problem for a stamp located on an elastic half-space is given below in a simpler way. In the following work (Akhazhanov, Omarbekova et al., 2020), a new method for obtaining an analytical solution to the problem of deflection a beam on an elastic foundation is considered. The elastic foundation is calculated using the deflection stiffness parameter, the paper shows a comparative analysis of the Winkler method and the proposed new method. In the work of the following authors (Akhazhanov et al., 2023; Akhazhanov, Bostanov et al., 2023) a new numerical method for calculating a beam on a two-parameter elastic foundation is proposed, the finite element method was used for calculation. The construction of a finite element stiffness matrix was presented.

Relevance. The construction of an analytical solution to the contact problem for a solid stamp on an elastic foundation is a great theoretical and practical interest in the field of modern construction mechanics of a deformable solid, however, it is associated with serious mathematical difficulties and is not always possible. Therefore, an urgent problem is the development and improvement of new, simple numerical and analytical methods for solving these problems. Goal. Development of a simplified method for calculating a solid stamp on a two-parameter elastic foundation.

Research methodology. The calculation for a solid stamp on an elastic foundation is considered. We apply the obtained solution to a solid beam (stamp) lying on an elastic half-plane. A stamp is installed on the elastic foundation, the following values are shown: the length of the stamp l , the length outside the stamp L , the height of the elastic foundation h , the evenly distributed load q_0 on the stamp (Figure 1).

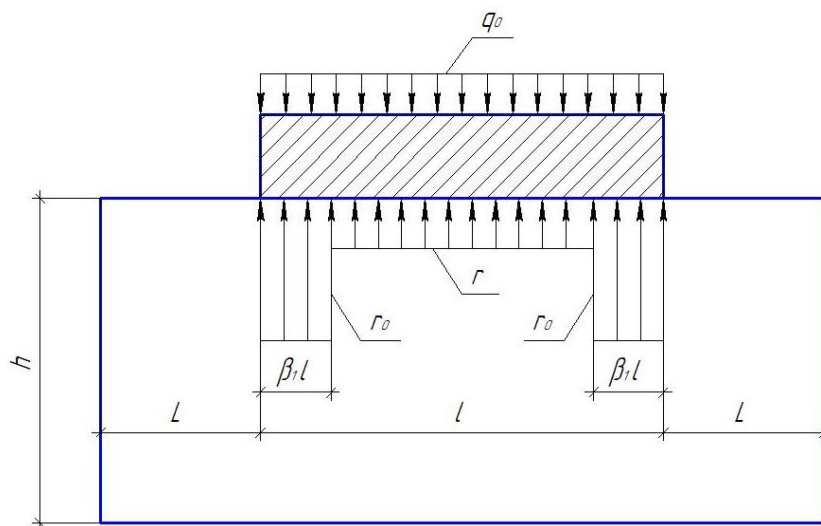


Figure 1. A solid stamp

Note – compiled by the author

In this case, the solution of the problem is completely determined by the formulas (Kasimov, Akhazhanov & Kassenova, 2024) only ($\beta=1$) is equal to (l is the length of the stamp).

$$\begin{aligned}
 W_* &= \frac{\gamma_1}{\gamma}, r = \frac{\pi(1-\nu^2)}{2} \frac{E}{l} \frac{\gamma_1}{\gamma}, \\
 \gamma &= 1 + \pi(1-\nu^2) \cdot \beta \left(\frac{L}{h}\right)^3, \\
 \gamma_1 &= \frac{2\beta l}{E} q_0 \left(\frac{L}{h}\right)^3
 \end{aligned} \tag{1}$$

The normal voltage is determined using the following expression:

$$\alpha(z_0) = e^{-kz_0}(1 + kz_0) = -\frac{6(1-\nu^2)}{k^3}(1 + kz_0)e^{-kz_0}$$

$$\sigma_3 = \frac{k^4\beta}{6\gamma} \alpha(z_0) \left(\frac{L}{h}\right)^3 \left(\frac{l}{h}\right) \cdot q_0 \quad (2)$$

The vertical displacement of the point of application of force on the half-plane (Akhazhanov, 2020):

$$f(z_0) = \left[1 + \frac{k(1+\nu)}{2}z_0\right] \cdot e^{-kz_0}$$

$$U_3(x_1, z_0) = f(z_0) \frac{2\beta l q_0}{E\gamma} \left(\frac{L}{h}\right)^3 \quad (3)$$

According to (1), the reactive pressure of the half-plane acting on the stamp is determined as follows ($\beta = 1$):

$$r = \frac{\gamma_{11}}{\gamma_{12}} q_0$$

$$\gamma_{11} = \pi(1 - \nu^2) \left(\frac{L}{h}\right)^3, \quad \gamma_{12} = 1 + \gamma_{11} \quad (4)$$

Resultant force of external and reactive pressure according to Flaman:

$$2R = \beta \cdot \ell(q_0 - r) \quad (5)$$

The reactive pressure r_0 on the edge of the stamp is determined using the formula (5):

$$R = \beta_1 r_0 l$$

$$\beta_1 r_0 l = \frac{q_0 l}{2} \left(1 - \frac{\pi(1-\nu^2)\left(\frac{L}{h}\right)^3}{1 + \pi(1-\nu^2)\left(\frac{L}{h}\right)^3}\right)$$

$$\beta_1 r_0 = \frac{q_0}{2\gamma_{12}} \quad (6)$$

$$r_0 = \frac{q_0}{2\gamma_{12}\beta_1}$$

The stamp draft at ($\beta = 1$) is determined by (1):

$$W_* = \frac{\gamma_{23}}{\gamma_{22}}$$

$$\gamma_{12} = 1 + \pi(1 - \nu^2) \left(\frac{L}{h}\right)^3 \quad (7)$$

$$\gamma_{23} = \frac{2l}{E} q_0 \left(\frac{L}{h}\right)^3$$

Methods for calculating the stamp can be found in the works of Sadovsky and Dinnik. But $r_0 = \infty$ is a disadvantage of their solutions. In the literature (Galín, 1953), the reactive pressure of the stamp (Fig. 1) under the action of a concentrated force is defined as:

$$r(x_1) = \frac{2P}{\pi l \sqrt{1 - \frac{4x_1^2}{l^2}}} \quad (8)$$

where: P – is the concentrated force.

If $x_1 = \frac{l}{2}$ (edge of the stamp), then $r\left(\frac{l}{2}\right) = \infty$. This is a disadvantage of this calculation. In our solution, the reactive pressure at the edge of the stamp is r_0 . If in (7) $\beta_1=0$, then $r_0=\infty$. In this case, our solution will also have a disadvantage, but the value of the reactive pressure r_0 will depend on the length (L) of the half-plane, which is deformed outside the stamp, and its thickness (h) and according to (8), the reactive pressure depends only on the length of the stamp. The stamp is often used in life: in construction – the foundation; in blacksmithing – the sternum, etc. Therefore, the value of the reactive pressure (7) is high.

Results and discussions. Below, consider calculations performed using a solid stamp on an

elastic two-parameter foundation. Using this method, it is possible to determine a draft, displacements of the ground and resistance of the soil foundation to external loads.

Determination of the mending on an elastic half-plane foundation under the stamp. Initial Data for calculation: stamp length $l=1\text{m}$, beam length parameter $\beta=0.01$, elastic foundation thickness $h=1\text{m}$, Poisson's ratio $\nu=0.3$, length of the foundation lying inside the stamp $L=1\text{m}$, a uniformly distributed load $q_0=5\text{kN/m}$, the soil foundation is homogeneous. The calculation was based on the length of the foundation lying inside the stamp. With various changes in the elastic modulus of the foundation, the values of the deflection function were determined in how it affected the foundation. The mending of the elastic foundation is calculated for given different modules of elasticity of the soil foundation $E1=0.01\text{Pa}$, $E2=0.05\text{Pa}$, $E3=0.1\text{Pa}$, $E4=0.5\text{Pa}$ (Table 1).

Table 1. Deflection function values

x (m)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
W1(x)	9.722	9.722	9.722	9.722	9.722	9.722	9.722	9.722	9.722	9.722	9.722
W2(x)	1.944	1.944	1.944	1.944	1.944	1.944	1.944	1.944	1.944	1.944	1.944
W3(x)	0.972	0.972	0.972	0.972	0.972	0.972	0.972	0.972	0.972	0.972	0.972
W4(x)	0.194	0.194	0.194	0.194	0.194	0.194	0.194	0.194	0.194	0.194	0.194

Note – compiled by the author

Where: $W1(x)$, $W2(x)$, $W3(x)$, $W4(x)$ – values of the deflection on an elastic half-plane foundation under the stamp. From the calculated values of the function, we see that as the modulus of elasticity of the soil foundation decreases, the value of the deflection function increases. As a result, the bearing property of the foundation shows a decrease, the elastic soil foundation tends to perceive heavy loads, affects its settlement and displacement. The values of the deflection calculations are displayed as a graph (Figure 2).

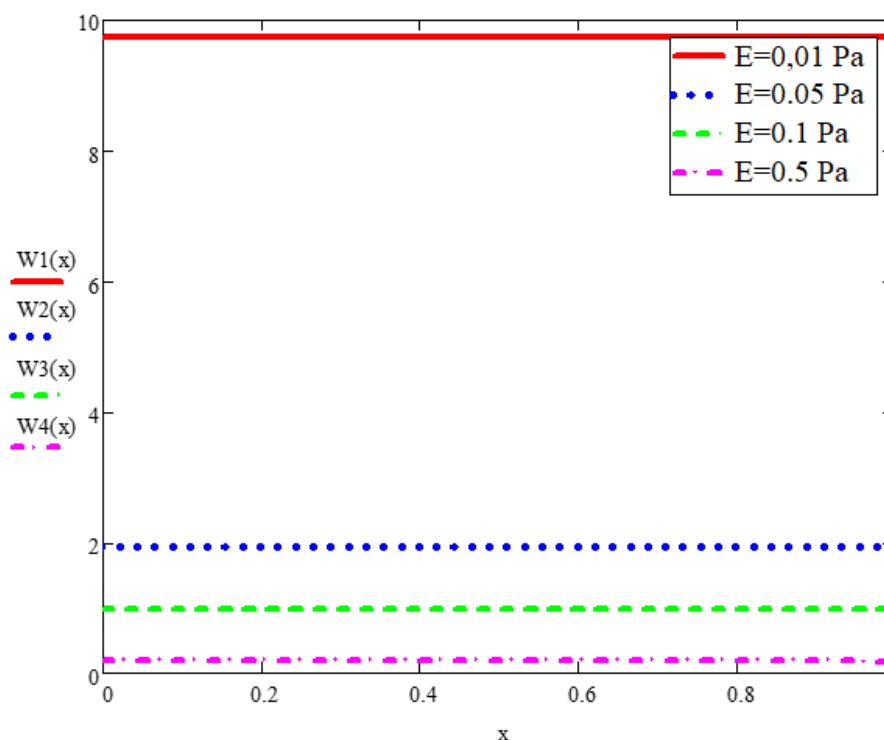


Figure 2. The deflection of the half-plane affecting on the stamp

Note – compiled by the author

Also in this paper, the change in the values of normal stress and vertical displacement is determined by changing the l/h ratio lying outside the stamp. Table 2 shows the maximum values of normal stress and vertical displacement at the specified intermediate parameters. As a result, the higher the length of the base lying outside the stamp, the lower the value of the normal stress, and the value of vertical displacement increases on the contrary. In addition, the values of normal stress and vertical displacement were determined by various parameters of the height of the elastic foundation. Carrying out these calculations makes it possible to conduct research on the entire length of the elastic foundation.

Table 2. Maximum values of normal stress and vertical displacement at specified intermediate parameters

L/h	$\sigma_3(z_0)$	$U_3(x_1, z_0)$
0,1	$-1,429 \cdot 10^{-4}$	$2 \cdot 10^{-4}$
0,5	-0,018	0,025
1	-0,139	0,194

Note – compiled by the author

The values of the results obtained from the calculation at different heights of the elastic foundation of vertical displacement were presented in the form of a diagram (Figure 3).

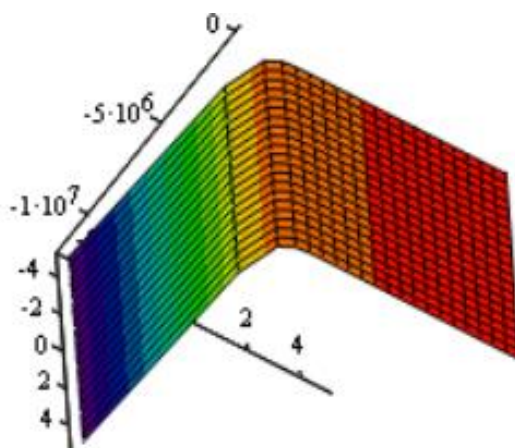


Figure 3. The vertical displacement of the elastic foundation

Note – compiled by the author

It is vital to consider the distribution of reactive pressure on the elastic half-plane, since the beam is calculated for uniformity from its solution. In this paper, the reactive pressure of the stamp is determined under the action of a concentrated load. In the calculation process, the beam length parameter was assumed to be $\beta_1=0.01$, because if $\beta_1=0$ is assumed, then the values of the reactive pressure will be infinite. The calculation of the reactive pressure of the elastic foundation and the parameters of the elastic body was determined by the formula (4). The reactive pressure at the edge of the stamp was determined according to the formula (6). All calculations were performed in the MathCAD program, the results are shown below.

$$\gamma_{11} := \pi(1 - \nu^2) \left(\frac{L}{h}\right)^3 = 2.857, \quad \gamma_{12} := 1 + \gamma_{11} = 3.857$$

$$r(x) := \frac{\gamma_{11}}{\gamma_{12}} \cdot q_0 = 3.7037901176958573132$$

$$r_0(x) := \frac{q_0}{2\gamma_{12}\beta_1} = 64.810494115207134339$$

$$R_1(x) := r_0(x) + \left(\frac{107}{30}x^4 - \frac{107}{15}x^3 + \frac{107}{30}x \right)$$

$$R_1(x) := \begin{cases} r_0(x) & \text{if } x < 0.1 \\ r(x) & \text{otherwise} \end{cases} \quad R_2(x) := \begin{cases} r_0(x) & \text{if } x > 0.9 \\ r(x) & \text{otherwise} \end{cases}$$

$$R(x) := R_1(x) + R_2(x)$$

The uniform forces of external and reactive pressure are shown in the form of a graph (Figure 4) with a high value of reactive pressure at the longitudinal edges of the elastic foundation. Also, as a result of the calculation, it was found that as the values of the x parameters in the stamp increase, the values of the reactive pressure on the elastic foundation approach the stamp.

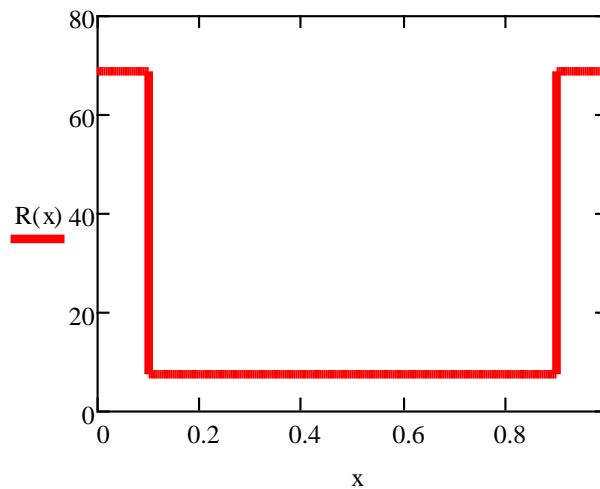


Figure 4. Reactive pressure of the elastic foundation

Note – compiled by the author

Conclusion. To summarize, simplified formulas for new calculations of a solid stamp on an elastic two-parameter foundation were considered. Using calculations, the values of ground sediment, vertical displacement, deflection and reactive pressure were determined. Using different elastic modulus in calculations, the values of the deflection function were compared, as a result of which the deflection value decreases as the elastic modulus increases. The values of vertical displacement and normal stress arising on the elastic foundation were changed taking into account the length lying outside the stamp. As a result, the value of the length lying outside the stamp decreases, the value of the vertical displacement decreases, and the value of the normal stress increases. In addition, this paper defines the values of the reactive pressure at the edge of the stamp and located horizontally, which occurs on an elastic foundation on which the stamp is installed. The values of the reactive pressure fluctuate depending on the distance of the regions on the elastic foundation. The system of solving equations is fully automated, and the calculation results are presented in the form of tables and plots, graphs.

Conflict of interest. The author(s) declare that there is no conflict of interest.

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elastic foundation).

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References

- Khryanina O.V. (2016). Sovmestnaya rabota gibkogo fundamenta s armirovannym osnovaniem: monografiya/ O.V. Khryanina. – Penza: PGUAS, -204s. ISBN 978-5-9282-1431-9
- Tsvei A.Yu. (2014). Balki i plity na uprugom osnovanii. Lektsii s primerami rascheta po spetsial'nomu kursu stroitel'noi mekhaniki: ucheb. posobie / A.Yu. Tsvei. – M.: MADI, – 96 s.
- Yaretskaya N.F. (2018). Contact Problem for the Rigid Ring Stamp and the Half-Space with Initial (Residual) Stresses. *Int Appl Mech*, vol. 54, 539–543, <https://doi.org/10.1007/s10778-018-0906-y>.
- Balabušić M., Folić B., Čorić S. (2019). Bending the Foundation Beam on Elastic Base by Two Reaction Coefficient of Winkler's Subgrade. *Open Journal of Civil Engineering*, vol. 9, 123-134, DOI: 10.4236/ojce.2019.92009.
- Bosakov S. V. (2019). Reshenie kontaktnoi zadachi dlya uzla opiraniya balochnoi sharnirno opertoj plity. *Nauka i tekhnika*, T. 18, № 4, 274-283, <https://doi.org/10.21122/2227-1031-2019-18-4-274-283>.
- Ledenov V.V., Monastirev P.V., Kulikov G.M., Plotnikova S.V. (2016). Raschetnye modeli dlya proektirovaniya konstruksii i zdaniy : monografiya dlya nauchnykh i inzhenerno-tekhnicheskikh rabotnikov, aspirantov, magistrantov i studentov stroitel'nykh spetsial'nostei. Tambov: Izd-vo FGBOU VO «TGTU», – 296 s. – 400 ekz. ISBN 978-5-8265-1660-7
- Yulong Li., Arutiunian A., Kuznetsova E., Fedotenkov G. (2020). Method for solving plane unsteady contact problems for rigid stamp and elastic half-space with a cavity of arbitrary geometry and location. *Incas Bulletin*, vol. 12, 99-113; <https://doi.org/10.13111/2066-8201.2020.12.S.9>.
- Arutyunyan A., Kuznetsova E., Fedotenkov G. (2020). Plane unsteady contact problem for a rigid stamp and an elastic half-space with a cavity. *Trudy MAI*, 113, 2-28, <https://doi.org/10.34759/trd-2020-113-02>.
- Lysandros P., Elias G. (2020). Elastic Settlement Analysis of Rigid Rectangular Footings on Sands and Clays. *Geosciences*, vol. 10, no. 12, 491, <https://doi.org/10.3390/geosciences10120491>.
- Bosakov S.V. (2023). Contact Problem for the End of Elastic Half-Strip. *Science & Technique*, 22(2), 127-130, <https://doi.org/10.21122/2227-1031-2023-22-2-127-130>.
- Akhazhanov S., Omarbekova N., Mergenbekova A., Zhunussova G., Abdykeshova D. (2020). Analytical solution of beams on elastic foundation. *International Journal of Geomate*, 19(73), 193–200, DOI: 10.21660/2020.73.51487.
- Akhazhanov S.B., Vatin N.I., Akhmediyev S., Akhazhanov T., Khabidolda O., Nurgoziyeva A. (2023). Beam on a two-parameter elastic foundation: Simplified finite element model. *Magazine of Civil Engineering*, 121(5), 12107, DOI: 10.34910/MCE.121.7.
- Akhazhanov S., Bostanov B., Kaliyev A., Akhazhanov T., Mergenbekova A. (2023). Simplified method of calculating a beam on a two-parameter elastic foundation. *International Journal of GEOMATE*, vol. 25, no. 111, 33–40, <https://doi.org/10.21660/2023.111.3898>.
- Kasimov A.T., Akhazhanov S.B., Kassenova A.N., Abdrakhmanova K.A. (2024). Zhartylai serpimdili negizdi shorylanfan kysh əserine zertteu. *Trudy universiteta*, no. 3, 189-196, DOI: 10.52209/1609-1825_2024_3_189.
- Akhazhanov S.B. (2020). Serpimdili negizdegi arqalyqty esepeteu ədisi : monografiya / S. B. Akhazhanov ; QR bilim zhəne ƣylym min-gi, Akad. E.A.Beketov atyndary Ƙaraƣandy memlekettik un-ti. - Ƙaraƣandy : ƘarMU baspasy, 166, ISBN 9789965-39-868-1.
- Galın L.A. Kontaknyye zadachi teorii uprugosti. – M.: Gostekhizdat, 1953. – 268 s.

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