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EFFECTIVE TOOLS THAT INFLUENCE THE DEVELOPMENT OF STUDENTS' RESEARCH ABILITIES IN MATHEMATICS

МАТЕМАТИКАДАН ОҚУШЫЛАРДЫҢ ЗЕРТТЕУ ҚАБІЛЕТІН ДАМЫТУҒА ЫҚПАЛ ЕТЕТІН ТИІМДІ ҚҰРАЛДАР

ЭФФЕКТИВНЫЕ ИНСТРУМЕНТЫ, СПОСОБСТВУЮЩИЕ К РАЗВИТИЮ ИССЛЕДОВАТЕЛЬСКИХ СПОСОБНОСТЕЙ ШКОЛЬНИКОВ ПО МАТЕМАТИКЕ

Abstract. *Currently, much attention is paid to the development of mathematical abilities, analytical and systemic thinking skills, the foundations of schoolchildren and students. Ensuring the continuity of educational programs of secondary and higher education is the "core" of successful education of schoolchildren to form research competence, instill search and justification skills, and a fundamental attitude towards mathematics. The practical significance of the work is that by transforming problems in higher mathematics, it is possible to bring school mathematics and mathematics closer together in higher educational institutions, thereby forming students' research search skills, the desire for knowledge outside the framework of the program. The analysis of the results obtained is reasonable and shows that it will make objective changes in the pedagogical activities of mathematics teachers in search of ways to improve in applied research.*

Keywords: *research skills; mathematical thinking; research abilities; approaches; geometric problems.*

Аңдатпа. *Қазіргі уақытта математикалық қабілеттерді, талдамалық және жүйелі ойлау дағдыларын, оқушылар мен студенттердің негіздерін дамытуға көп көңіл бөлінуде. Орта және жоғары білім беру бағдарламаларының сабақтастығын қамтамасыз ету зерттеу құзыреттілігін қалыптастыру, іздестіру және негіздеу дағдыларын, математикаға іргелі қарым-қатынасты үйрету үшін оқушыларды табысты оқытудың «өзегі» болып табылады. Жұмыстың практикалық маңыздылығы жоғары математика бойынша тапсырмаларды өзгерту арқылы жоғары оқу орындарында мектеп математикасы мен математикасын жақындастыруға болады, сол арқылы студенттерде зерттеу іздестіру дағдыларын, бағдарлама шеңберінен тыс білімге ұмтылысты қалыптастыруға болады. Алынған нәтижелерді талдау негізді болып табылады және ол қолданбалы зерттеулерде жетілдіру жолдарын іздестіруде математика мұғалімдерінің педагогикалық*

қызметіне объективті өзгерістер енгізетінін көрсетеді.

Түйін сөздер: *зерттеу дағдылары, математикалық ойлау, зерттеу қабілеттері, тәсілдері, геометриялық міндеттері.*

Аннотация. *В настоящее время большое внимание уделяется развитию математических способностей, навыков аналитического и системного мышления, основ школьников и студентов. Обеспечение преемственности образовательных программ среднего и высшего образования является «стержнем» успешного обучения школьников для формирования исследовательской компетентности, привития навыков поиска и обоснования, фундаментального отношения к математике. Практическая значимость работы заключается в том, что путем преобразования задач по высшей математике можно сблизить школьную математику и математику в высших учебных заведениях, формируя тем самым у студентов навыки исследовательского поиска, стремление к знаниям вне рамок программы. Анализ полученных результатов является обоснованным и показывает, что он внесет объективные изменения в педагогическую деятельность учителей математики в поисках путей совершенствования в прикладных исследованиях.*

Ключевые слова: *исследовательские навыки, математическое мышление, исследовательские способности, подходы, геометрические задачи.*

Introduction. Paragraph: use this for the first paragraph in a section, or to continue after an extract.

This paper suggests approaches to the formation and development of research skills in high school students. It should be noted that methods of solving geometric problems, geometric interpretation of the problem are one of the main approaches to the formation of mathematical thinking in students. It is interesting that the approaches of ensuring the continuity of school mathematics and mathematics in higher education within the framework of the transformation of tasks, focusing on the knowledge of the school course of mathematics can be taken as one of the main methods. This will allow instilling research skills, fundamental knowledge and conclusions of remarkable properties of geometric objects. The concept of "differential equations" describes mathematical models of some geometric problems related to the tangent to the curve. The transformation of geometric problems leading to the compilation of differential equations can be easily applied to determine the particular properties of second-order curves and their derivation based on mathematical knowledge on the basis of a school course. Using the example of one problem, it is possible to characterize and prove many interesting properties of second-order curves defined using a tangent. In this regard, it is important to study second-order curves as curves with an extended mechanical application. Determination of particular properties of second-order curves, which are proved analytically only on the basis of a function tangent to the graph, have important applications in determining the trajectories of moving celestial bodies, the construction of projection apparatuses, solar installations. The purpose of the work is to develop analytical abilities and instill a research character in students by transforming problems of higher mathematics into properties using methods of mathematical analysis, geometry. Within the framework of the formulated goal , the following tasks are set and solved:

– Conversion of applied problems to standard mathematical problems of the school course;

– Analysis of the impact of the applied methodology on the development of students' research abilities.

Review of the research. In the works of [1-4] conducted research in the field of transformation of mathematical knowledge based on geometric interpretations. These transformations have their advantages in determining the applicability of a particular method associated with the visualization of a geometric object, modeling. The importance of visualization of these objects is that they make their reasonable contribution to the development of mathematical abilities of students, to the formation of mathematical thinking skills in the development of

algorithms and the acquisition of the ability of accurate reasoning. In these studies, visual representation is used to increase the indicators of mathematical representation abilities, which symbolize a mathematical statement. In this direction, the application of project-based learning is explained and justified as one of the special methods that have an impact on the mathematical representation of the student from both psychological and mathematical points of view. The use of mathematical representation and correct connection will help students to concretize mathematical ideas and link one concept with another concept in order to form a view of mathematics as an integration in general.

In the works of [5-6], the importance of understanding problems, drawing diagrams, correctly reading diagrams of mathematical understanding and solving mathematical problems is noted to improve the understanding of visual thinking abilities. This, in turn, can affect the development of research skills, data interpretation skills in students, which is very important for understanding the modern world and scientific literacy.

In the works of [7-9], the authors of the researchers considered and expanded the elements of modeling theory as a basis for research and development in the field of mathematics and natural science education, as approaches to represent the connections between spatial vision and mathematical understanding and thinking. This is due to the fact that knowledge in natural science, mathematics and everyday life is interpreted with the creation and use of mental models, which in turn will allow you to describe and understand the nature and quantity of spatial and mathematical skills.

[10] presents the authors' conclusions about the awareness of mathematical learning of students, that it is an essential factor and contributes to various aspects of teaching practice.

In the works of [11-12] the authors investigated and noted the abilities of students about the implementation of translation from one form of representation to another form of representation, the so-called fundamental abilities to build conceptual and mathematical thinking. Studies have shown a qualitative result about the process of translation by students from verbal representation to graphical representation at each stage of translation. These results fit well into the competence for the formation of research skills in students.

In the works of [13-14] the authors revealed that mathematical representations are the main tool, also an indicator of mathematical knowledge and abilities. To strengthen mathematical knowledge, you need to have a deep mastery of mathematical language, mathematical communication. But the research results show that not many students possess mental models of representation and interpretation by them, and the authors have suggested ways to improve.

In the work of [15-17] the authors proposed a solid foundation and foundation for future theoretical work in the field of mathematical education based on representations and symbol systems and this process is closely tied to professional practice, as well as knowledge and skills necessary for work.

The articles [18-19] argue that advanced mathematical thinking should be understood as thinking in advanced mathematics. This follows from the fact that the formation of mathematical thinking entails the formation and development of all types of other thinking, including systemic, analytical. The results show that everyone involved in the educational and educational process was selected and assigned tasks that should lead to the development of students' thinking abilities.

In the works of [20-23] discussed that conventional teaching methods alone cannot affect the improvement of mathematical creativity and thinking skills needed by a modern child at the present time. We need more advanced and modern methods, mixed methods of interdisciplinary research for the formation and development of critical mathematical thinking skills in students. The

basis of all this is a deep mathematical understanding and mathematical knowledge. In this regard, the authors offer a number of recommendations for improving mathematical education starting from primary school and continuing in secondary school.

In the works of [24-26] show that creativity, as one of the keys to success in the developing world economy, is also a manifestation of fundamental literacy based on mathematical understanding of the process, which is absolutely necessary in the 21st century. Also in mathematics, it is important to develop creativity or creative thinking, since creativity is an integral part of mathematics. However, the limitation of students' creative abilities leads to a set of skills that are automated with selection, sometimes not subject to any logic. To avoid this, the authors propose to consider the tasks and questions of PISA that teachers need to master and remember in order to form higher-order thinking skills, as the main implementers of ideas for the development of creativity, creative abilities in students.

[27] present research methods for realistic mathematical education based on the application of learning approaches aimed at developing competencies using Sketchpad Geometry. Although the data analysis method used is descriptive analysis, however, the result shows that the implementation of realistic mathematical education using the Geometry notebook in proportions can improve the critical and creative thinking of students.

In the works of [28-29] presents research among mathematics teachers on the subject of awareness of the influence of students' views, beliefs and views on mathematics on their study and use of mathematics. Ideas are proposed for introducing novelty into the vocational guidance training of students, in determining the role of mathematics in solving engineering problems and engineers' perception of the status of mathematics in comparison with other resources and limitations.

Materials and methods of research. Consider the following geometric problem: Find curves for which the area of the triangle formed by the tangent, the ordinate of the tangent point and the abscissa axis is a constant value equal to a^2. The transformed formulation of this problem is aimed at obtaining interesting properties of second-order curves associated with the definition of conic sections. The transformed properties are determined by visual representation of conic sections. Thus, it is possible to ensure the continuity of secondary and higher education aimed at the formation and development of analytical and systemic thinking, instilling the research character of a young scientist.

Property 1. The area of a triangle formed by a tangent and its intersection with the asymptotes of a hyperbola $x^2 - y^2 = a^2$, obtained by the intersection of a cone given by $x^2 = y^2 + z^2$ an arbitrary plane equation $z = \pm a$, is equal to the same constant number a^2 (Figure 1).

After analyzing the data, comparing the facts, considering possible options, the student will calculate the area of the triangle and draw conclusions based on the data obtained.

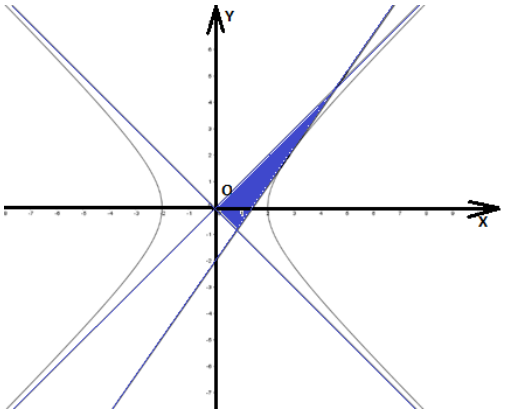

Figure 1. Graphical representation of the property 1

Property 2. The area of a triangle formed by a tangent and its intersection with an asymptote and a perpendicular from the tangent point to the asymptotes of a conic section $x^2 - y^2 = a^2$ obtained by $x^2 = y^2 + z^2$ the intersection of a cone given by an arbitrary plane equation $z = \pm a$ is equal to the same number $\frac{a^2}{4}$ (Figure 2).

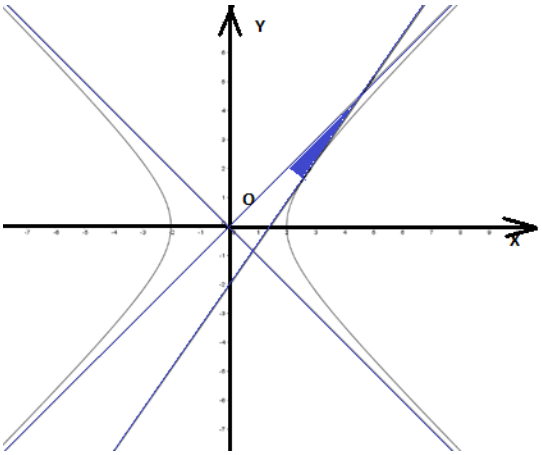

Figure 2. Graphical representation of the property 2

To make a decision, students mentally make a detailed plan. When proving this property, they will have a feathering on the previous property to obtain complete information about the nature of the curves.

Property 3. The coordinates of the intersection points of the tangent at the point with the asymptotes for the hyperbola $x^2 - y^2 = a^2$ obtained by the intersection of the cone given by the equation $x^2 = y^2 + z^2$ with an arbitrary plane $z = \pm a$, will be equal to the same values $P(x_0 + y_0; x_0 + y_0)$ and $Q(x_0 - y_0; y_0 - x_0)$ (Figure 3).

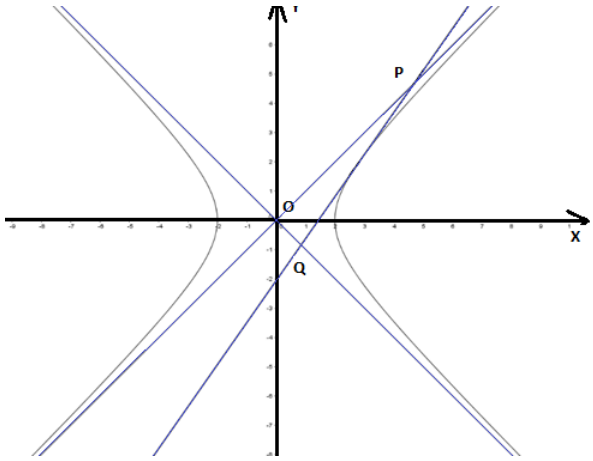

Figure 3. Graphical representation of the property 3

This property will allow the student to see the whole problem, thereby recognize the curve completely by the example of their properties and describe the algorithm of their actions:

decompose it into several components,

analyze each element,

highlight positive and negative points,

prioritize;

choose the most appropriate option or method of solving the problem.

Property 4. A triangle formed by tangents drawn to any symmetric points relative to the axis of symmetry and a straight line passing through the ordinate of the points of a parabola (Figure 4) $y^2 = 2px$ obtained by the intersection of a cone given by an equation $x^2 = y^2 + z^2$ with an arbitrary plane parallel to one of the generators of the cone with a conventional designation $z = x - p$ has the coordinates of the center of the circle of nine Euler points equal $\left(\frac{p}{4};0\right)$.

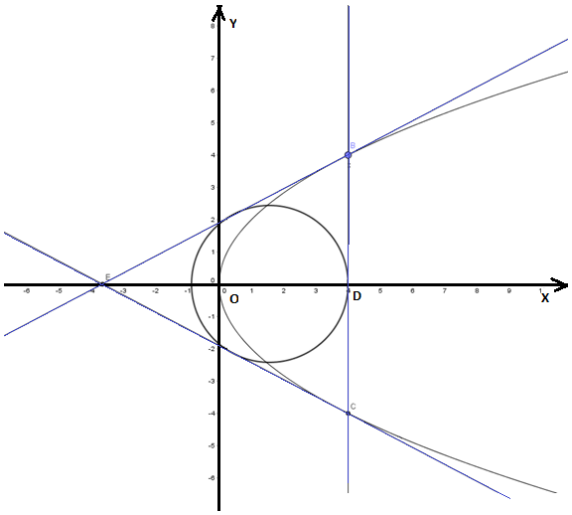

Figure 4. Graphical representation of the property 4

To prove the property, we make up the equation of two tangents to the parabola at points symmetric with respect to the axis of symmetry. Next, to determine the triangle formed by the condition of the property, you need to find the coordinates of the intersection point of two tangents. This property differs from other properties in that the student will intentionally look for a connection with previous properties to prove this property. In this case, the student decomposes the information into components, structures the data and finds out their relationship to each other. This approach allows the student to make the best of all possible decisions.

Results and discussion. To analyze the impact of the applied methodology on the development of research abilities of students, the following components of research competence were identified:

K1. The motivational-value component determines the motivational-value attitude to research activities

−shows cognitive activity;

−shows a desire for self-improvement;

−striving for creative research activities;

−readiness to identify the problem, forecast;

−the desire to work in a team of researchers.

K2. The cognitive-content component determines knowledge and understanding about the elements of the research:

−knows the sequence of the study;

−knows research methods;

−knows how to work with information;

−knows goal setting, planning, evaluation of results;

−knows communicative ways of expression.

K3. Activity component the level of formation of the student's research skills:

−plans research activities;

−defines the goal, objectives, topic, predicts hypothesis, research;

−makes an overview of sources;

−argues, expresses;

−conducts experiments, proves, justifies;

K4. The evaluation component determines the formation of the student's evaluation skills:

−ranks by significance, optimality, efficiency;

−high level of self-control;

−reflection;

−making critical judgments;

−analyzes, synthesizes, compares, compares.

The weighting coefficients of the components of research competence and indicators were determined using expert assessments of school mathematics teachers (10 people). Initially, the experts were asked to rank the listed knowledge, skills, abilities, personality qualities, etc. according to the degree of significance for the student's readiness for research activities. The survey obtained the original data matrix, elements of which are rated expert ranks characterizing the importance of a component in the process of formation of the research competence: Weighting factors for each of the components: K1=0,23, K2=0,25, K3=0,26, K4=0,26.

Conclusion. To identify the consistency of the opinions of 10 experts as a criterion was defined as the coefficient of concordance from the formula

$$W = \frac{S}{S_{max}} \Rightarrow W = 0{,}76.$$

The obtained concordance coefficient indicates a high degree of consistency of experts in ranking criteria. The indicator of the level of formation of students' research abilities depends on the levels of development of its components.

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