



ЭЛЕКТРОНИКА. РАДИОТЕХНИКА  
ЭЛЕКТРОНИКА. РАДИОТЕХНИКА  
ELECTRONICS. RADIO ENGINEERING

DOI 10.51885/1561-4212\_2021\_3\_53  
MPHTI 47.61.01

**A. Kalizhanova<sup>1,3</sup>, W. Wojcik<sup>1,2</sup>, M. Kunelbayev<sup>1</sup>, A. Kozbakova<sup>1,4</sup>, L. Cherikbayeva<sup>1,5</sup>,  
Zh. Aitkulov<sup>1,5</sup>, Zh. Amirgaliyeva<sup>1</sup>**

<sup>1</sup>Institute of Information and Computational Technologies CS MES RK, Almaty, Kazakhstan

<sup>2</sup>Lublin Technical University, Poland

<sup>3</sup>Almaty University of Power Engineering and Telecommunications, Almaty, Kazakhstan

<sup>4</sup>Almaty University of Technology, Almaty, Kazakhstan

<sup>5</sup>Al-Farabi Kazakh National University, Almaty, Kazakhstan

E-mail: kalizhanova\_aliya@mail.ru\*

E-mail: waldemar.wojcik@pollub.pl

E-mail: murat7508@yandex.kz

E-mail: ainur79@mail.ru

E-mail: lyailya\_sh@mail.ru

E-mail: jalau@mail.ru

E-mail: zh.amirgaliyeva@gmail.com

## ANALYSIS OF BRIDGE VIBRATION, USING FIBER-OPTIC BRAGG SENSORS WITH TILTED GRATING

### КӨЛБЕУ ТОРЛЫ БРЭГГ ОПТИКАЛЫҚ-ТАЛШЫҚТЫ ДАТЧИКТЕРІНІҢ ҚОЛДАНЫЛУЫМЕН КӨПІРЛЕРДІҢ ВИБРАЦИЯСЫН ТАЛДАУ

### АНАЛИЗ ВИБРАЦИИ МОСТОВ С ИСПОЛЬЗОВАНИЕМ ВОЛОКОННО-ОПТИЧЕСКИХ ДАТЧИКОВ БРЭГГА С НАКЛОННОЙ РЕШЕТКОЙ

**Abstract.** The article herein presents methodology of identification and analysis of selected dynamic characteristics of bridge construction displacement, applying fiber-optic Bragg sensors with tilted grating. In the work herein there is described the main principle of sensors operation, based on fiber Bragg grating. As well, there have been defined vibration regimes, using analysis of time series with high discretization frequency. By means of corresponding mathematical models (filters) and spectral analysis there have been revealed and described construction's vibration frequencies, which have been compared with a theoretical model describing the dynamic behavior of the bridge. The given model has been checked at automobile and pedestrian bridge. By means of developed mathematical and experimental models, the data has been processed and vibration regimes upon bridge construction vibration were defined.

**Keywords:** Fiber-optic sensors, displacement, frequency analysis.

**Аңдатпа.** Мақалада көлбеу торлы Брэгг оптикалық-талшықты датчиктерін қолдана отырып, көпір құрылымының деформациясының таңдалған динамикалық сипаттамаларын анықтау және талдау әдісі берілген. Берілген жұмыста талшықты Брэгг торына негізделген датчиктердің негізгі жұмыс принципі сипатталады. Сондай-ақ, дискретизациялаудың жоғары жиілігімен уақытша қатарларды талдауды қолдану арқылы вибрация режимдері анықталды. Сәйкес математикалық модельдерді (сүзгілерді) және спектрлік талдаулардың көмегімен, көпірдің динамикалық күйімен сипатталған теориялық модельмен салыстырылған құрылымның тербеліс жиілігін анықтауға және сипаттауға болады. Берілген модель автомобиль және жаяу

жүргізіліп, көпшілігі арқылы сынақтан өткізілді. Құрылған математикалық және эксперименттік модельдердің көмегімен мәліметтер өңделді және көпір конструкцияларының вибрациялануындағы вибрация режимдері анықталды.

**Түйін сөздер:** Оптикалық-талшықты датчиктер, деформация, жиіліктік талдау.

**Аннотация.** В статье представлена методика идентификации и анализа выбранных динамических характеристик деформации мостовой конструкции с использованием волоконно-оптических датчиков Брэгга с наклонной решеткой. В данной работе описывается основной принцип работы датчиков на основе волоконной брэгговской решетки. Также были определены режимы вибрации с использованием анализа временных рядов с высокой частотой дискретизации. С помощью соответствующих математических моделей (фильтров) и спектрального анализа были выявлены и описаны частоты колебаний конструкции, которые сравнивались с теоретической моделью, описывающей динамическое поведение моста. Данная модель была проверена на автомобильном и пешеходном мосту. С помощью разработанной математической и экспериментальной модели были обработаны данные и определены режимы вибрации при вибрации конструкции моста.

**Ключевые слова:** Волоконно-оптические датчики, деформация, частотный анализ.

*Introduction.* Knowledge of dynamic characteristics of bridge construction behavior becomes, currently, increasingly significant. They are mainly caused by the wind and objects movement on the construction (pedestrians, bicyclists, transport means). They make an impact on construction's resonance behavior, which brings to deformation of construction's dynamic. For safety constructions operation it is necessary to simulate displacement and control during load tests. Moreover, there is a needed of long-term monitoring of the structural health by applying appropriate methodologies.

In works [1,2,3] researchers contributed much into calibration of structural numerical model of sustainable and safe bridge operation.

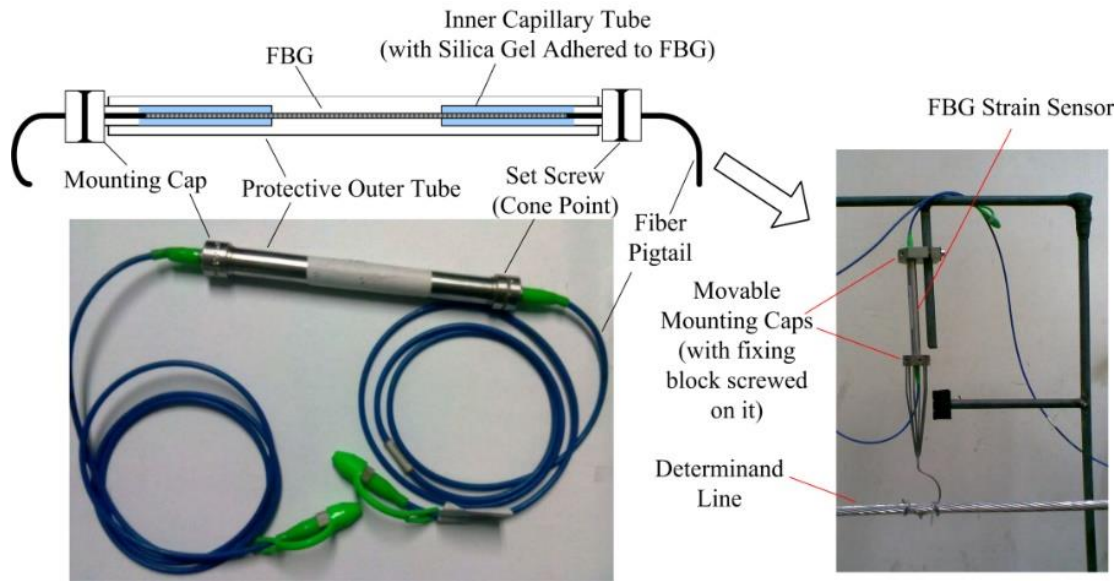
Different technologies can be used for that. One of them is fiber-optic sensors. Fiber-optic sensors operate according to the principle of changing physical properties of light transfer in glass fibers. Glass fiber mechanical displacement causes change in light refractive index. It might be used for measuring structural displacements.

In distinction from electrical sensors, where displacement causes change in output signal from electrical current, fiber Bragg grating makes an impact on refraction signal change. In works [4,5,6,7] there has been computed high sustainability of signal transmission for big distances, multiplexing and comparable accuracies with electrical sensors, which allow more frequent use of those sensors for monitoring structures health.

The work [8] shows, that remote sensing block and tensometer have crucial importance for icing monitoring scheme. In the construction it was decided to use FBG tensometers on the basis of silica in order to avoid temperature impact, as it is known for its high thermal stability, such as small coefficient of thermal expansion and compact solid structure. FBG tensometer structure is shown in Fig. 1. It has tubular incapsulation. In outer protective steel tube there are coaxial internal capillary tubes, containing FBG. FBG is adhered to internal tubes, and glue is usually made of materials for sensitization to strain with small Young's modulus, such as silica gel, epoxy resin and acrylic polymer. Internal tubes have two mounting caps at the edges, from which fibers are taken out.

Fiber-optic sensors represent the fragment of optic fiber, subjected to certain modification. Having used optical fibers as sensitive elements, there is no impact on the result of electromagnetic fields measuring, electromagnetic irradiation side effect, channels cross noises, no problems connected with earth loops and displacement strains in junction points of dissimilar conductors, moreover there is substantial increase in electric safety, there are no problems of arcing and sparking. Fiber-optic sensors have high resistance to hazardous environment; small sizes

and weight; high mechanical strength; stability to elevated temperatures, vibration, etc.; high speed of data transmission. Furthermore, fiber-optic sensors might be used in explosive atmosphere due to their absolute explosion safety. They are chemically inert, have simple construction and high reliability. Fiber Bragg gratings are different due to their differences in structure and photosensitivity of used fibers, additionally to peculiarities of recording conditions and lasers, by means of which recording is executed. Recording peculiarities are exposition time and recording dynamics, i.e., irradiation density. There are number of various techniques of constructing sensor systems, based on Bragg gratings. The simplest scheme of fiber-optic sensor is presented in Figure 2.



**Figure 1.** Construction of FBG tensometer

Signal from the source is reflected by sensing element. Reflection wavelength is fixed by analyzer block. As a rule, an analyzer represents narrow band spectrometer.

Reflected wavelength is known as Bragg wavelength and assigned as

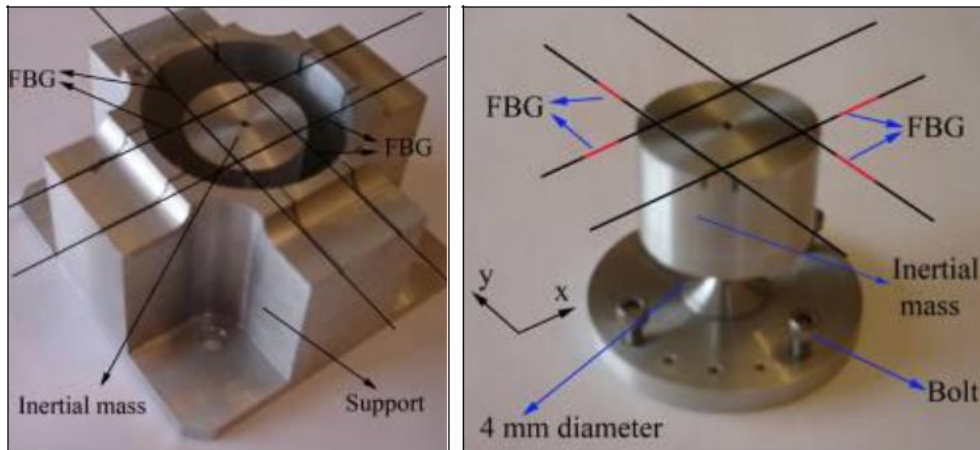
$$\lambda_B = 2n\Lambda \quad (1)$$

where  $n$ -effective refractive index of fiber core and  $\Lambda$  – index modulation period. Both  $n$  and  $\Lambda$  depend on temperature and displacement, consequently, Bragg wavelength is sensitive both to displacement and temperature [9].

*Displacement measurement principle.* In equation (1) reflected wavelength is up to Bragg condition. Temperature changes or fiber mechanical displacement cause grating deformation and change of effective refractive index and distance between the gratings. It makes an impact on displacement of reflected signal wavelength. Wavelength changes are dependent on temperature changes and displacement, which is used for qualitative assessing the grating deformation [10].

Bottom of central alumina detail has cylindrical form with 4mm diameter, as shown in the Figure 2. At availability of external acceleration, the sensor inert mass, having in the upper part 50mm diameter, freely moves in broadside direction. Inert mass can freely move in two inde-

pendent directions (x and y). Inert mass stretches and compresses optic fibers, which brings to measured Bragg wavelength displacement, which might be connected with applied external acceleration on accelerometer base. External acceleration along the base axis direction (x or y) brings to extension of one FBG and compression of another one, located opposite. External acceleration results from differential between Bragg displacements of two gratings wavelengths, located in each direction. That method increases acceleration measurement sensitivity twice.



**Figure 2.** Construction of implemented accelerometer (on the left) and scheme of central alumina detail of implemented accelerometer (on the right)

The article [11] generalizes existing fiber-optic schemes of readings, which were transformed from silica glass optic fibers into monocrystalline sapphire optic fibers (SFs- sapphire fibers). Single-point temperature measurements have been developed, carried out at 1900° C extremely high level, which has been achieved, using the first order SFs, manufactured with femtosecond laser, phase mask and Talbot interferometer system. As another example, distributed high temperature measurements have been developed, fulfilled at temperatures up to 1000° C, and have been implemented, using single mode sapphire fibers with 11 mm spatial resolution. The article [12] researched the characteristics of temperature and deformation measurements for two fiber Bragg gratings (FBG) of high order (3<sup>rd</sup> and 4<sup>th</sup> orders). FBGs have been fabricated from single-mode fiber SMF-28 (SMF-Single-Mode Fiber) without hydrogen loading, applying femtosecond laser on the basis of point-by-point direct recording method. Annealing operation has been investigated at temperature change up to 900°C. Being offered FBGs own potential for measuring high temperature, measuring deformation with high resolution. The article [13] offers the sensor with fiber Bragg grating (FBG), which can measure temperature, vibration and deformation. That sensor's architecture has been elaborated and optimized by means of theoretical analysis and finite-element method (FEM). For sensors fabrication there have been used single-mode FBGs, recorded with femtosecond laser (FS-FBG) and analyzed detection characteristics. Outcomes show, that the sensor thereof, on FS-FBG basis can withdraw temperature up to 1100°C. In the article [14] there has been developed a sensor system, which might constantly measure temperature up to 873 K and detect cracks on metallic tubes. In work [20] there has been elaborated a tilted FBG sensor with a tilt angle equal to 5°. Sensor's temperature sensitivity at 75 Pm/°C for the core mode within temperature range has shown the value from 298 K to 348 K.

*Materials and methods of research.* The sensor with tilted fiber Bragg grating has been developed. Application of the developed technique to data processing has been implemented at automobile bridge in Kazakhstan, Almaty region on the river Talgar.

Analysis of bridge constructions vibration. The main aim of bridge dynamic deformation vibration analysis is defining the vibrations regime and frequency. Every vibration regime is described with the form of structural displacement at standard frequency of its iteration.

Vibration study is, mainly, based on spectral analysis and signals processing.

In case of bridges displacement there is most frequently used fast Fourier transformation (FFT). FFT is defined as in [16]

$$X_x(f) = \sum_{k=0}^M \gamma_x(k)w(k)e^{-i2\pi fk/f_x} \quad (2)$$

where  $\gamma_x(k)$  – auto correlation function and  $w(k)$  – spectral window function [16].

Alternative is Walsh method application, which uses FFT algorithm.

To compute general spectral density there used spectra normalized values [17]:

$$X_i^{norm}(k) = \frac{X_i(k)}{\sum_{k=0}^{k-n} X_i(k)} \quad (3)$$

Final ANPSD (averaged normalised power spectral densities- ANPSD) is computed as arithmetical average of all normalized periodograms:

$$X_{priem}(k) = \frac{1}{p} \sum_{i=1}^p X_i^{norm}(k) \quad (4)$$

ANPSD describes distribution of spectral density of each time series, which can provide global idea about dynamic properties of the structure under control. Relevant frequencies identification is fulfilled by means of statistical criterion of Fisher periodicity sequence [18].

Next step is filtration of time series in frequency domain, applying band-pass filtering. An appropriate algorithm is Butterworth filter, which is the filter with irregular pulse characteristic. That filter has longer frequency characteristic, but, on the other hand, the filter has minimal attenuation in transmission band and pulsation in filtration band [19].

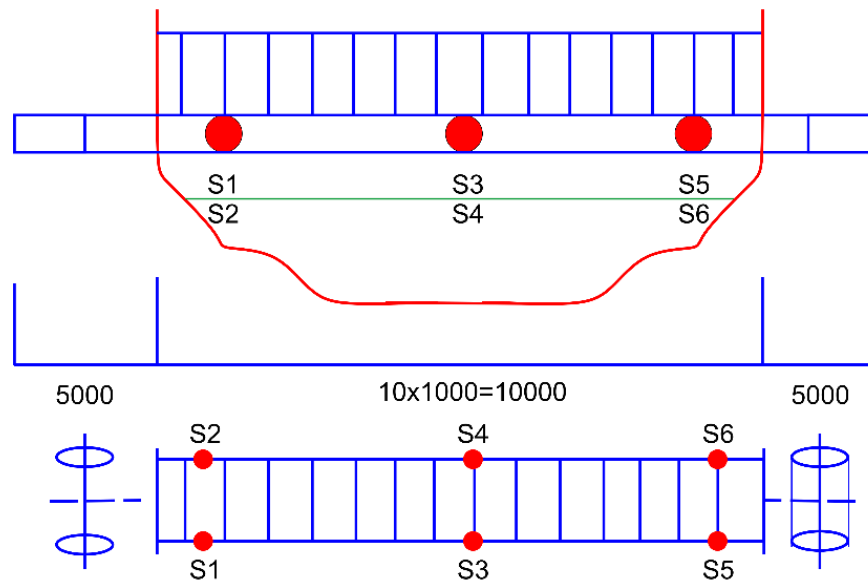
Cross spectral analysis of two-time series (signals) is used for cross correlation and time delay between them. Cross spectral density of two-time series might be assessed by means of FFT cross-correlation function as follows:

$$X_{xy}(f) = \sum_{k=0}^M \gamma_{xy}(k)w(k)e^{-i2\pi fk/f_x} \quad (5)$$

where  $\gamma_{xy}(k)$ -mutual correlation function and  $w(k)$  – spectral window function [20].

Based on the analysis of frequency amplitudes and phase delays between the signals being measured, in every measuring point there can be defined construction's vibration regime. Vibration regime identification accuracy is reached with help of a number of used sensors. A large number of sensors can provide more reliable definition of vibration regime.

For experimental measurements there have been used 6 sensors, located in three cross sections, where structural deformations are typical of the construction under control. The measurement was carried out by accelerometers and the configuration of measurement points on the bridge (i.e. the distribution of the accelerometers along the bridge) represents Figure 3.



**Figure 3.** Configuration of measuring points on the bridge

FBG sensors with tilted fiber Bragg grating are relatively sensitive to high temperature, which causes temperature drift of measuring signal. Temperature data is also applied to measure acceleration temperature compensation and to additionally interpret the temperature changes in the structure. To register temperature changes there have been used contact temperature sensors Pt1000/TG7. Those sensors were located in the same places as other sensors.

*Experimental measurements.* Experimental trials of FBG sensors with tilted fiber Bragg grating and developed mathematical models of vibration analysis were conducted on April 22, 2021. Measurements were carried out under several types of construction loading, such as the movement of automobiles and pedestrians.

Experiments were carried out every 5 minutes. Data registration frequency of FBG sensors with tilted grating was 200 Hz, which was conditioned with the requirements to reach higher accuracy degree of relative movements and occurrence of structural deformations significant values, exceeding 10 Hz.

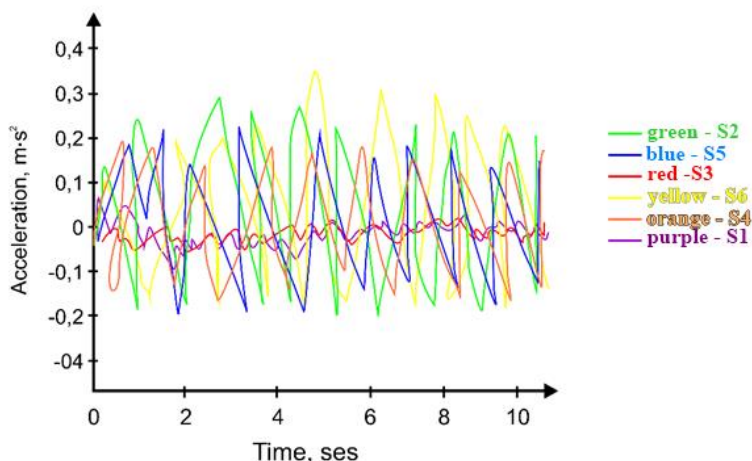
*Data processing and analyzing.* Data processing is based on applying the mathematical model. Measurements were carried out under specific dynamic load movement of one automobile and a man near measuring points S1 and S2 (Fig. 4).

**Results and Discussion.** Measured acceleration is illustrated in Fig. 4. There might be seen sufficiently higher maximum acceleration values in points S1 and S2, compared with other measuring points. A minimal level of maximum acceleration has been registered in points S3 and S4. Points S5 and S6 demonstrate lower values of maximum accelerations and time delay at the level of half period of vibrations sideward points S1 and S2, which corresponds to the 1st vibration regime.

Green, blue, red, yellow, orange, violet.

Figure 4 shows measured accelerations in measuring points.

Figure 4 shows measured accelerations in measuring points. The first stage of data processing, as a rule, is specifying construction's sufficient vibration frequencies in each measuring point.

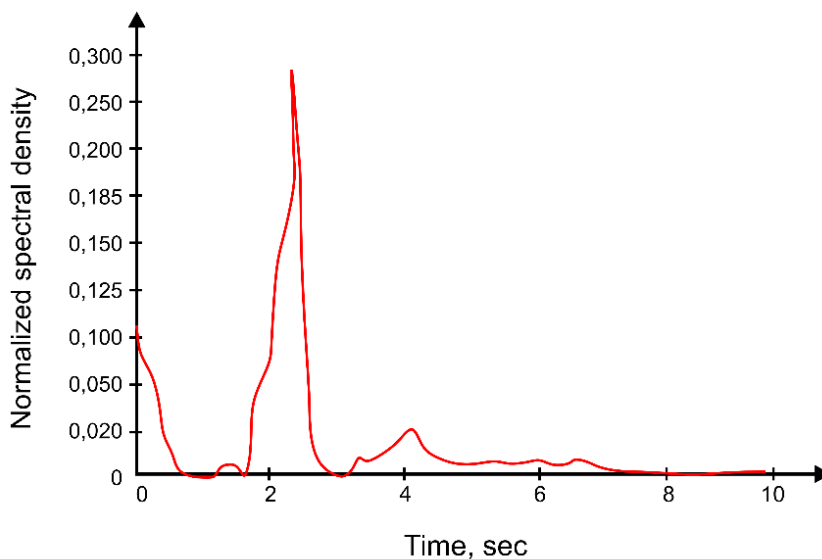


**Figure 4.** Measured acceleration in measuring points

Table 1 presents frequencies and amplitudes of every point.

**Table 1.** Measured frequencies of the construction's 1st vibration regime (1,42 Hz)

Measuring point	Frequency, Hz	Normalized amplitude	Difference, %
S1	1.325	0.2832	0.28
S2	1.322	0.2665	0.25
S3	1.325	0.0245	0.25
S4	1.321	0.0067	0.25
S5	1.325	0.1702	0.25
S6	1.321	0.2009	0.25
ANPSD		0.1989	0.25



**Figure 5.** ANPSD vibrations of bridge construction

Figure 5 shows ANPSD vibrations of bridge construction. Measured frequencies relatively well correspond to model frequencies. Differences between measured frequencies amount up to 0,29 %, which can be assessed as excellent correspondence.

The next stage is to define general characteristics of vibrations regimes in construction's different points by means of cross-spectral analysis.

Table 2 shows the total normalized bridge amplitudes and phase delays at the measurement points for the 3 sensors S1, S3, and S5 located on the structure.

**Table 2.** General normalized amplitudes and phase delays in measuring points S1, S3 and S5

1 <sup>st</sup> measuring point	2 <sup>nd</sup> measuring point	Normalized amplitude	Phase delay
S1	S3	0.2234	5.6611
S2	S5	0.3856	175.3456
S5	S5	0.2356	172.4567

**Table 3.** General normalized amplitudes and phase delays in measuring points S2, S4 and S6

1 <sup>st</sup> measuring point	2 <sup>nd</sup> measuring point	Normalized amplitude	Phase delay
S2	S4	0.1656	4.3478
S2	S6	0.2987	176.5678
S4	S6	0.1856	175.2345

As we see from the Table 3 the phase delay between measuring points, located on one and the same bridge, might reach minimal values (to 2°), which represent signals time delay at the level of 0,01 s. Those values well correspond to the experiment, settled for vibration regime at vertical bend without torque vibrations.

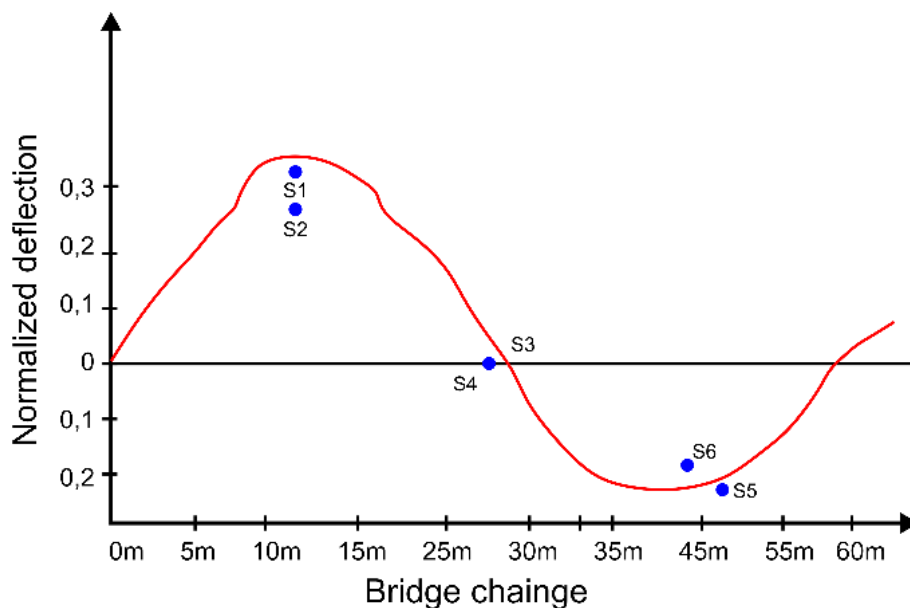
Table 4 presents construction deformation normalized values in each point.

**Table 4.** Construction normalized displacement for the 1<sup>st</sup> vibration regime

Measuring point	Normalized amplitude
S1	0.2546
S2	0.1857
S3	-0.0057
S4	-0.0567
S5	-0.1843
S6	-0.1589



Figure 6 shows the bridge structure displacement.



**Figure 6.** Bridge structure displacement, defined by experimental measurements, modeling outcomes

As it is seen from Figure 6, the bridge construction both from the left and right sides corresponds to the mathematical model. Despite the fact, that bridge's structure form is generated with a small number of sensors (points), it well matches the selected sensors' model (Fig.6). To update the accuracy of specifying the construction's vibration regime using acceleration measurement, it is necessary to supplement the measuring system with a bigger quantity of sensors. The outcome of that might be the structure's displacement, defined with higher resolution and reliability. Analysis of the dynamic behavior of bridge structures in recent years has been heavily impacted. The differences between the calculated and measured vibration modes of the structure, measured at regular (constant) intervals, make it possible to determine the possible imperfection of the structure. This can be traced in more detail using other common techniques used for bridge diagnostic behavior. The article presents a methodology for identifying and analyzing the selected dynamics of the deformation characteristics of a bridge structure using fiber-optic acceleration sensors [21]. This article presents a vibration measurement system based on low-frequency cantilever accelerometers with a Fiber Optic Bragg Array (CFA) for a suspension bridge. Each accelerometer has a cantilever terminated beam specially designed to achieve uniform sensitivity over the 0-4 Hz frequency range, a suitable detection range for vibration analysis. Infield trials, seven CFAs were installed at specific locations on the deck of a 110 m suspension bridge for synchronous multi-point vibration measurements. Reflection spectra of the CFA array were recorded and processed using a pseudo-high resolution scheme to improve the signal quality and measurement accuracy [22]. The advantage of applied technology is in the fact, that a sensor can detect and effectively monitor, in a short period, different defects of the bridge. Obtained outcomes show measurements stability and higher accuracy, compared with meas-

urements, than measurements using traditional optical sensors. Fiber-optic measuring techniques possess several important advantages, such as resistivity to environmental effects (variable temperature or electromagnetic field), relatively simple possibility to place the sensors on the construction, and simultaneous measurements in many points. These include remote sensing, ease of installation, no corrosion, and lower maintenance costs.

*Conclusion.* The article herein presents the possibility of defining the bridge construction's displacements using FBG with tilted grating under dynamic loading of the construction. In the work, there have been elaborated the methodology and mathematical model, based on spectral analysis and signals processing to define vibrations frequencies and analyze vibrations regimes. Practical usage of FBG with tilted grating and application of developed data processing methodology has been implemented on the automobile bridge in Kazakhstan, Almaty region on river Talgar. Experimental measuring outcomes correspond to developed construction and might be used for its calibration. Considerable contribution of the work herein into forecasting probable failures and bridge structure utilizing experimental works constitutes vibration regimes specification and frequency change.

*Acknowledgments.* This work is supported by a grant from the Ministry of Education and Science of the Republic of Kazakhstan within the framework of the Project GF AP09259547 «Development of a system of distributed fiber-optic sensors based on fiber Bragg gratings for monitoring the health of building structures», Institute of Information and Computational Technologies, CS MES RK.

#### References

1. Kohut, P., Holak, K., Uhl, T., Krupiński, K., Owerko, T., Kuraś, P. 2012. Structure's Condition Monitoring Based on Optical Measurements. In Key Engineering Materials, Vol. 518, pp. 338-349, DOI 10.4028/www.scientific.net/KEM.518.338
2. Braun, J., Štroner, M. 2014. Geodetic Measurement of Longitudinal Displacements of the Railway Bridge In INGENIO 2014. Prague: CVUT in Prague, 2014, vol. 1, p.231-236. ISBN 978-80-01-05469-7.
3. Wenzel, H. (2009). Health Monitoring of Bridges. John Wiley & Sons, Ltd. 2009. 643 pp. ISBN 978-0-470-03173- 5.
4. Glišić, B. Inaudi, D. (2007). Fibre Optic Methods for Structural Health Monitoring. [online]. ISBN 978-0470-06142-8.
5. Guan, B. O. Tam, H. W. Liu, S. Y. (2004). Temperature Independent Fiber Bragg Grating-Tilt Sensor. [online]. Available at: <<http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=>>.
6. Honglei, G. Gaozhi, X. Nezih, M. Jianping, Y. (2011). Fiber Optic Sensors for Structural Health Monitoring of Air Platforms. In MDPI Open Access Publishing. [online]. 2011, vol. 11, no. 4. Available at: <[www.mdpi.com/1424-8220/11/4/3687/pdf](http://www.mdpi.com/1424-8220/11/4/3687/pdf)>. ISSN 1424-8220.
7. Zhang, X. Z. (2006). R&D of Various FBG Sensors for Practical Application in Infrastructures (Dissertation Thesis). Harbin Institute of Technology, 2006.
8. Min Z., Yimeng X., Zhiguo Z., Qiguan C. (2014). Design and Experiment of FBG-Based Icing Monitoring on Overhead Transmission Lines with an Improvement Trial for Windy Weather. In MDPI Open Access Publishing. [online]. 2014, vol. 14, no. 12. Available at: .
9. Sylex. (2014). Advantages of FBG sensors. [online]. Available at: [http://www.sylex.sk/fileadmin/user\\_upload/web/products/Technology/Advantages%20of%20FBG%20sensors.pdf](http://www.sylex.sk/fileadmin/user_upload/web/products/Technology/Advantages%20of%20FBG%20sensors.pdf)
10. Antunes, P. Travanca, R. Rodrigues, H. Melo, J. Jara, J. Varum, H. André, P. (2012). Dynamic Structural Health Monitoring of Slender Structures Using Optical Sensors. In MDPI Open Access Publishing. [online]. 2012, vol. 12, no. 5. <<http://www.mdpi.com/1424-8220/12/5/6629/htm>>.
11. Zhu C., Gerald R.E., Huang J. Progress towards sapphire optical fiber sensors for high-temperature applications. IEEE Trans. Instrum. Meas., 69 (11) (2020), pp. 8639-8655
12. Zhang Y., Qiao D., Zhu Y., Jiang P. High order fiber Bragg grating fabricated by femtosecond laser pulses for high-sensitivity temperature and strain sensing. Optik, 222 (2020), Article 165423.

13. Yao K., Lin Q., Jiang Z., Zhao N., Tian B., Peng G.D. Design and analysis of a combined FBG sensor for the measurement of three parameters. *IEEE Trans. Instrum. Meas.*, 70 (2021), pp. 1-10.
14. He J., Hu C., Hu D., Wang A. High temperature all fiber non destructive multi parameter sensing system with consistent performance. *Opt. Lett.*, 45 (7) (2020), pp. 1722-1725
15. Kipriksiz S.E., Yücel M. Tilted fiber Bragg grating design for a simultaneous measurement of temperature and strain. *Opt. Quantum Electron.*, 53 (1) (2021), pp. 1-15
16. Cooley, J. W. & Tukey, J. W. (1965). An algorithm for the machine calculation of complex Fourier series. *Mathematic Computation*. 19 (90). pp. 297-301.
17. Welch, P. D. (1967). The Use of Fast Fourier Transform for the Estimation of Power Spectra: A Method Based on Time Averaging Over Short, Modified Periodograms. *IEEE Transactions on Audio Electroacoustics*, pp. 70–73.
18. Wenzel, H. (2009). *Health Monitoring of Bridges*. John Wiley & Sons, Ltd. 2009. 643 pp. ISBN 978-0-470-03173- 5.
19. Trauth, M. H. (2010). *Matlab Recipes for Earth Sciences*. 3. vydanie. Springer – Verlag, 2010. 336 p. ISBN 978-3-642- 12762-5.
20. Bracewell. R. (1965) *Pentagram Notation for Cross Correlation. The Fourier Transform and Its Applications*. New York : McGraw-Hill, pp. 46-243, 1965.
21. I. Lipták, A. Kopáček, J. Erdélyi, P. Kyrinovič (2016) *Vibration analysis of bridges using fiber optic*. proceedings/2016/2016\_03\_JISDM.
22. K-S Lim, M. Zaini, Zhi.Ch. Ong, F. Abas, M. Salim *Vibration Mode Analysis for a Suspension Bridge by Using Low-Frequency Cantilever-Based FBG Accelerometer Array* *IEEE Transactions on Instrumentation and Measurement* DOI: 10.1109/TIM.2020.3018578.